1. $E(X|Y) = h(Y) \to$ function of Y
2. $E\,E(X|Y) = E(X)$ — Adam's Law (Law of Total Expectation)
3. $E\{[X - E(X|Y)]h(Y)\} = 0$
4. $\mathrm{Cov}(X - E(X|Y), h(Y)) = 0$

} $X - E(X|Y)$ — uncorrelated with any function of Y

5. $E(h(Y)\cdot X|Y) = h(Y)\cdot E(X|Y)$ — Take out evidence
6. $E(Y/X) = E(Y)$ if $X \perp Y$
7. $\mathrm{Var}(Y) = E(\mathrm{Var}(Y/X)) + \mathrm{Var}(E(X|Y))$ — EVVE's law (Law of total Variance)

Solving Nested expectation methodology

→ Smallest subset wins.

Method 1

eg 1.) $E[E[X|Y]/Y,Z] = E[X|Y]$

smaller subset $\{Y\}$ selected in answer

Intuition: $E(X|Y)$ – Think of taking expectation of X but not considering the variability of component of X that varies with Y
– When Y is evidence, then $E(X|Y)$ represents the expectation of (X) as a function of Y that doesn't take into account the variance component of X that varies due to Y.

Method 2 Intuition

$E(X|Y) = h(Y) \to$ represents average component of X that varies independent of Y.

$E[h(Y)/Y,Z]$

here Y is again evidence, so this variability of X as Y varies is again held (not averaged) $= E(X|Y)$

Intuition $E(E(X|Y)) = E(X) \to$ Here, the outer expectation is not holding any evidence, so variability of X even as a component of Y is averaged resulting in true $E(X)$.

eg 2.) $E\,E[X|Y]$

method 1 $\to = E[E(X|Y)/\phi] = E(X|\phi) = E(X)$

smaller set selected

eg 3) $E(E(X|Y,Z,W)/Y,Z)$ Method 1 $E(X|Y,Z)$

eg 4) $E(E(X|Y,Z)|Y) = E(X|Y)$

M1

$E[E(X/Y)\cdot g(Y)] = E[E[E(X|Y)\cdot g(Y)]/Y]$

$= E[g(Y)\cdot E(E(X|Y))] = E[g(Y)\cdot X]$

$E(E(X|Y,Z)/Y) =$

Solving questions with product inside expectation $\rightarrow$ Take one more expectation

$\Omega = E[Z\, E(X|Y)] =$

× a) $E(XZ)$

✓ b) $E[X \cdot E(Z|Y)]$

× c) $E(X) \cdot E(E(Z|Y))$

× D) $E[E(X|Z) \cdot E(Z|Y))$

Solution

$E(Z \cdot E(X|Y)) = E(E[Z \cdot E(X|Y)]/Y) = E[E(X|Y) \cdot E(Z|Y)]$

$\rightarrow E[X \cdot E(Z|Y)] = E(E(X|Y) \cdot E(Z|Y))$

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eg) $E[E(X|h(Y)) \,|\, Y] = E[X|h(Y)]$ T/F $\boxed{\text{true}}$

$h(Y) \subseteq Y$

less info   more info

M I

Q) $Cov(N,K) = Cov(N, E(K|N))$ T/F

We know $(K - E(K|N))$ has zero covariance with any function of N

ie $Cov(K - E(K|N), N) = 0$

$Cov(K,N) = Cov(K - E(K|N) + E(K|N), N) = Cov(K - E(K|N), N) + Cov(E(K|N), N)$

$$\Rightarrow \boxed{Cov(K,N) = Cov(E(K|N), N) = Cov(K, E[N|K])}$$